Choosing a Radar Algorithm to Use as a Proxy for E

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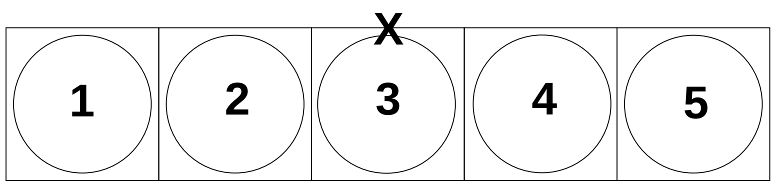
1 Box size — How much is enough?

Question: How much cloud (horizontally) do you have to include to capture "most" of the variability in **E***?*

Based on balloon soundings, the charges inside a typical thunderstorm are arranged in 500 m- to 1 km-thick layers. We have gridded the radar data into 1 km cubes. Because **E** obeys superposition, we can do the same for charge. Assume that each 1 km radar box contains a 1 km spherical charge. The airplane is flying through at point "×" .



The field at the surface of a sphere is $E = Q/4\pi\epsilon_o r^2$, where r is measured from the center of the sphere. Call this E_0. That is the contribution to **E** from sphere 3 at point ×.

- The centers of spheres 2 & 4 are 2 radii from the center of sphere 3, so the distance to point × is $r\sqrt{2^2 + 1^2} = r\sqrt{5}$, so each contributes $E = Q/4\pi\epsilon_o(r\sqrt{5})^2 = E_0/5$, and their total contribution is $(2/5)E_0$, or 40%.
- For spheres 1 & 5, the contribution is $(2/17)E_0$ or 12%.
- For spheres 0 & 6 (not shown), the contribution is $(2/37)E_0$ or 5%.

Thus it seems if we include cloud more than 3 boxes away from the aircraft in our radar algorithm, we are including cloud that contributes $< 5\%$ to **E.** However, between the aircraft and radar, we may have 1–2 km difference in geolocation. So we have to include another 1–2 boxes, giving us a domain size of 4–5 boxes on either side of the aircraft. This argues in favor of a 9×9 or 11×11 box size, and it argues *against* the 21×21 box size.

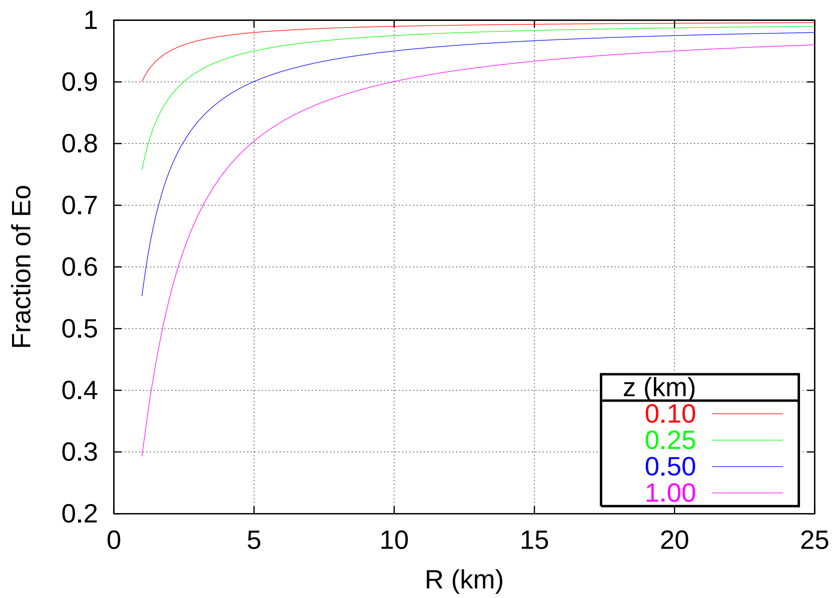
1.1 But how much is too much?

Let's try to do the same calculation in a continuous model, without discretizing the charge into spheres. The easiest geometry is to calculate **E** above a disc. Skipping details here, the field a distance z above a disc of radius R is

$$E = \frac{\sigma}{2\epsilon_o}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) = E_0\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right),$$

where this time $E_0 = \sigma/2\epsilon_o$, the field from an infinite flat plate.

Keeping z fixed and letting R grow, here are some plots of $E(R)$ for various values of z:



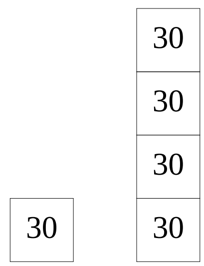
So how does our previous calculation compare? Realize that the first calculation was one dimensional, i.e., for a line of charges. This calculation is 2-D. Look at the graph for $R = 5\,\text{km}$. For the 2 smaller values of z, we have captured $\geq 0.95E_0$ by $R = 5\,\text{km}$. For the 2 large values of z, we have captured 90% and 80% of E_0.

But is that "good enough"? If we increase R to 10 km, the above fractions increase to 95% and 90%, i.e., an increase of 5% and 10%, respectively. To obtain these slight gains, we have had to increase our domain area by a factor of 4 — or by 300%.

Again, $R = 5\,\text{km}$ (11×11) seems to be a very reasonable upper limit on box size.

2 To Average or not to Average. . . ?

The problem with averaging is that it throws away depth information. For example:



Both of these average to 30 dBZ. But the deeper cloud is much more likely to be electrified. Thus, averaging is a *bad* idea.

3 Conclusion

Because of the above-mentioned reasons, a box size of 11×11 seems to be the largest reasonable size, and smaller might well work better. The problem with using too large a box is that one may be including cloud that is too far away to be relevant to **E.**

Also, for reasons given above, a column sum is a better indicator of clouds that are likely to electrify. Our analysis showed that using a column sum produced a better correlation between radar reflectivity and measured **E**. Indeed, the animations generated show a very nice correlation between $|\mathbf{E}|$ and the 1×1 column sum of reflectivity. This is what we have previously called "integrated reflectivity above 0°C."

In summation, we are in favor of smaller box sizes and *definitely against* averging. Further, it is *essential* that we use a column sum algorithm as our proxy.